# Enhancing the Understanding of Math Formulas using Visual Design Patterns

Group 10: Sebastian Bock,
Sijia Guo, Kristina Hörrmann

Media Computing Group
RWTH AACHEN UNIVERSITY

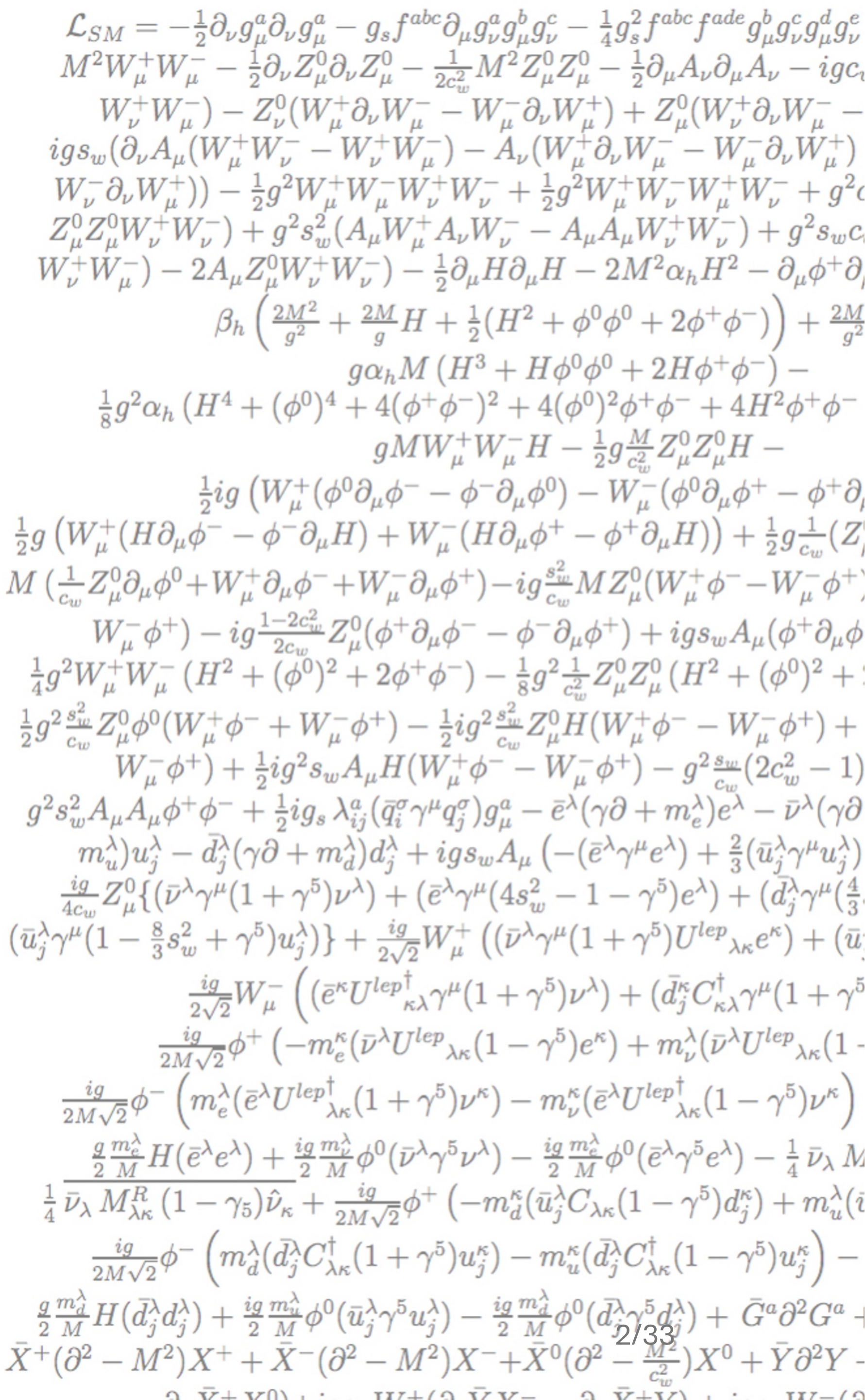
“Mathematics is important but boring”

Kislenko et al. (2007)

“Mathematics is difficult to understand” (71%)

“I am not able to understand the meanings of mathematical expressions” (70%)

Waswa, et al. (2023)



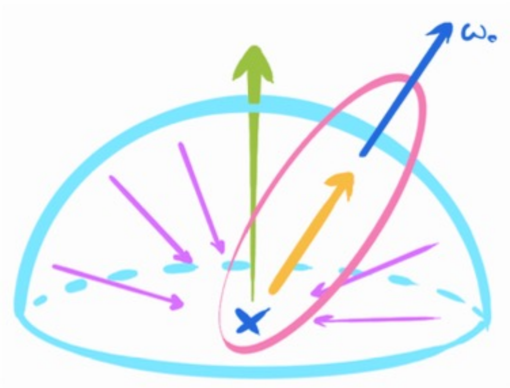
# Math Augmentation: How Authors Enhance the Readability of Formulas using Novel Visual Design Practices

Andrew Head
andrewhead@allenai.org
Allen Institute for AI
United States of America

Amber Xie
amberxie@berkeley.edu
UC Berkeley
United States of America

Marti A. Hearst
hearst@berkeley.edu
UC Berkeley
United States of America

$$L_0(\mathbf{x}, \omega_0) = L_e(\mathbf{x}, \omega_0) + \int_\Omega f_r(\mathbf{x}, \omega_i, \omega_0)\, L_i(\mathbf{x}, \omega_i)\, (\omega_i \cdot \mathbf{n})\, \mathrm{d}\omega_i$$



To find the light towards the viewer from a specific point, we sum the light emitted form such point plus the integral within the unit hemisphere of the light coming from a any given direction multiplied by the chances of such light rays bouncing towards the viewer and also by the irradiance factor over the normal at the point.

Note that incoming light is also computed by that very formula, which makes this exhaustingly recursive.

**Figure 1: A formula from computer graphics, visually embellished to improve its readability, from [5] (CC BY-NC-SA 4.0).** One author from our interview study created this formula and the accompanying colorized diagram and text to teach readers of his blog how to implement the formula in source code. Like many of the formulas analyzed in this paper, this one makes use of color to draw attention to conceptually important expressions in the formula, and to help a reader visually link those expressions to complementary diagrams and prose. Contents of the blog post (formula, prose, and diagram) have been rearranged in this figure to emphasize the formula.

## ABSTRACT

## CCS CONCEPTS

$$a = \frac{v - u}{t}$$

Acceleration is defined as the rate of change in velocity to the change in time.

$$a = \frac{v - u}{t}$$

Final velocity

Initial velocity

Acceleration

$$a = \frac{v - u}{t}$$

Time taken

**RQ** How do augmentations affect reading experiences?

**RQ** What is the maximum number of augmentations that can be added before it becomes overwhelming?

$$\varphi = P(y, g(f(z, e(u)), h(a)), g(e(u), h(e(z))))$$

First-order Logic

$$\varphi = P(y, g(f(z, e(u)), h(a)), g(e(u), h(e(z))))$$

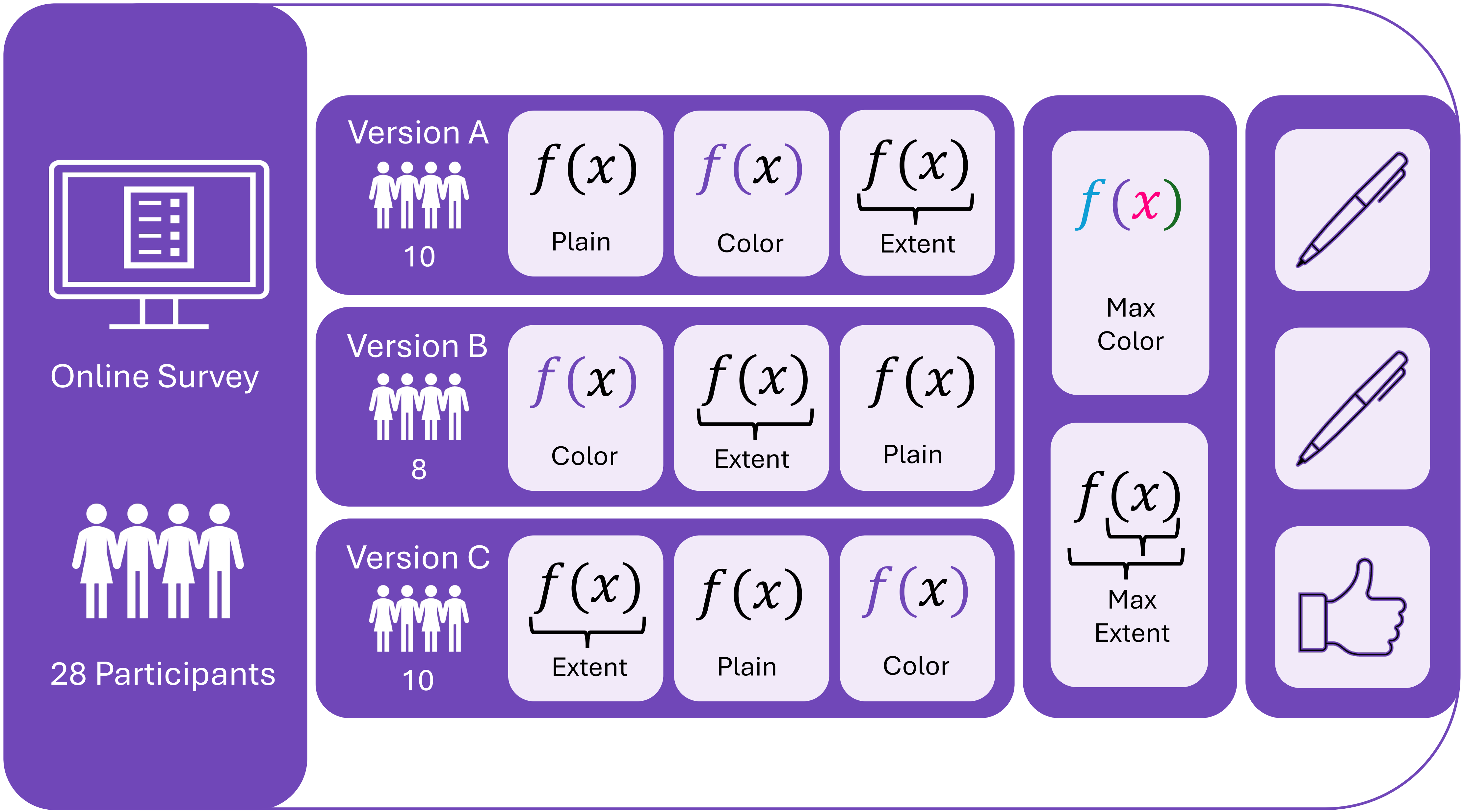
First-order Logic

$$\varphi = {\color{blue}P(}y{\color{blue},}{\color{purple}g(}{\color{orange}f(}z{\color{orange},}{\color{green}e(}u{\color{green})}{\color{orange})}{\color{purple},}{\color{teal}h(}{\color{red}a}{\color{teal})}{\color{purple})}{\color{blue},}{\color{purple}g(}{\color{green}e(}u{\color{green})}{\color{purple},}{\color{teal}h(}{\color{green}e(}z{\color{green})}{\color{teal})}{\color{purple})}{\color{blue})}$$

Color Augmentation

$$\varphi = P\overbrace{(y, \underbrace{g\overbrace{(f\overbrace{(z, e(u))}), h\underbrace{(\ a\ )})}, \underbrace{g\overbrace{(e(u)}, h\underbrace{(\overbrace{e(z)})})})}$$

Extent Augmentation


Online Survey
28 Participants
Version A
10
Plain
Color
Extent
Version B
8
Color
Extent
Plain
Version C
10
Extent
Plain
Color
Max Color
Max Extent

**RQ** How do augmentations affect reading experiences?

**H1** Mathematical formulas with augmentations are rated as easier to read compared to those without.

**H2** The use of colors is more effective than extents in improving readability and understanding.

$$\varphi = P(y, g(f(z, e(u)), h(a)), g(e(u), h(e(z))))$$

What is the arity of *P*? *

Your answer

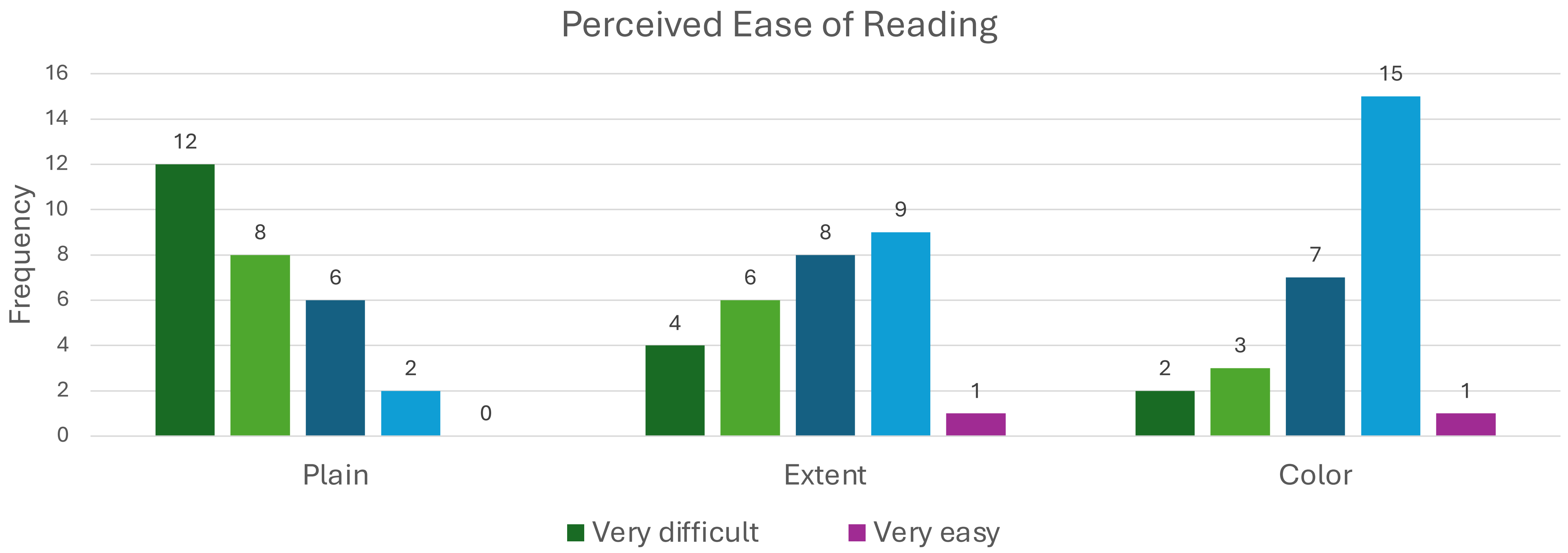
How easy was this formula to read? *

| | 1 | 2 | 3 | 4 | 5 | |
|---|---|---|---|---|---|---|
| Very difficult | ○ | ○ | ○ | ○ | ○ | Very easy |

**H1** Mathematical formulas with augmentations are rated as easier to read compared to those without.

**H2** The use of colors is more effective than extents in improving readability and understanding.

# How easy was this formula to read?




There was a **significant difference** between how easy people perceived the formulas to read ($X^2(2) = 24.5; p < 0.05$).

# How easy was this formula to read?

Post-hoc Analysis using Wilcoxon's Matched-Pairs Signed Ranks Test:

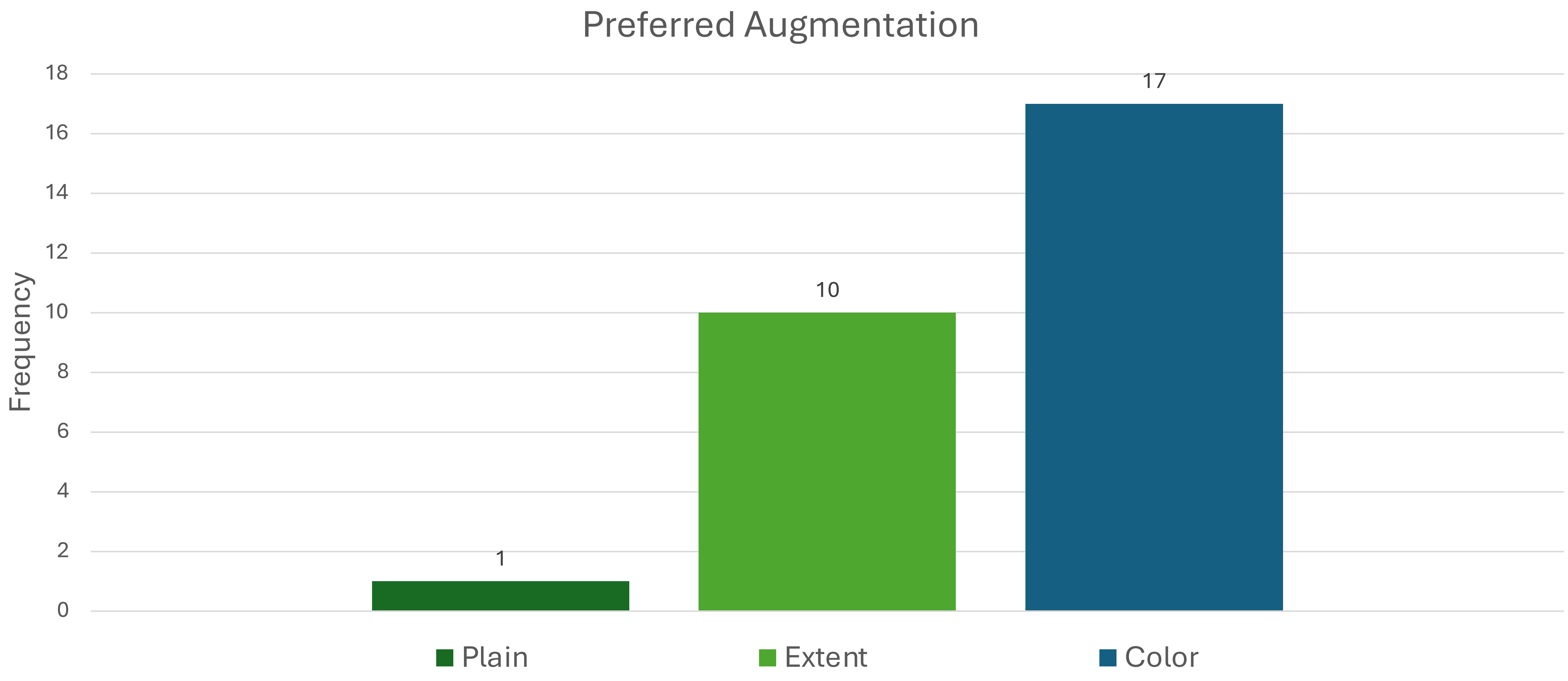
| H | Augmentation Type | | Direction | p-Value | T-Value |
|---|---|---|---|---|---|
| H1 | Color | Plain | Greater | $p < 0.05$ | 4.0 |
| | Extent | Plain | Greater | $p < 0.05$ | 13.0 |
| H2 | Color | Extent | Greater | $p < 0.05$ | 139.5 |

# How easy was this formula to read?

Post-hoc Analysis using Wilcoxon's Matched-Pairs Signed Ranks Test:

| H | Augmentation Type | | Direction | p-Value | T-Value |
|---|---|---|---|---|---|
| H1 | Color | Plain | Greater | $p < 0.05$ | 4.0 |
| | Extent | Plain | Greater | $p < 0.05$ | 13.0 |
| **H2** | Color | Extent | Greater | $p < 0.05$ | 139.5 |

# Which augmentation do you prefer?



**RQ** What is the maximum number of augmentations that can be added before it becomes overwhelming?

**H3** There is a threshold for how many colors can be added.

**H4** There is a threshold for how many extents can be added.

Color

Size 6

Size 16

Size 34

Extent

For the following formula, please select which version you prefer the most.

Option 1:

$$\varphi_7 = P(y, g(f(z, e(f(u)), h(a))), g(e(u), h(e(z))))$$

Option 2:

$$\varphi_7 = P(y, g(f(z, e(f(u)), h(a))), g(e(u), h(e(z))))$$

Option 3:

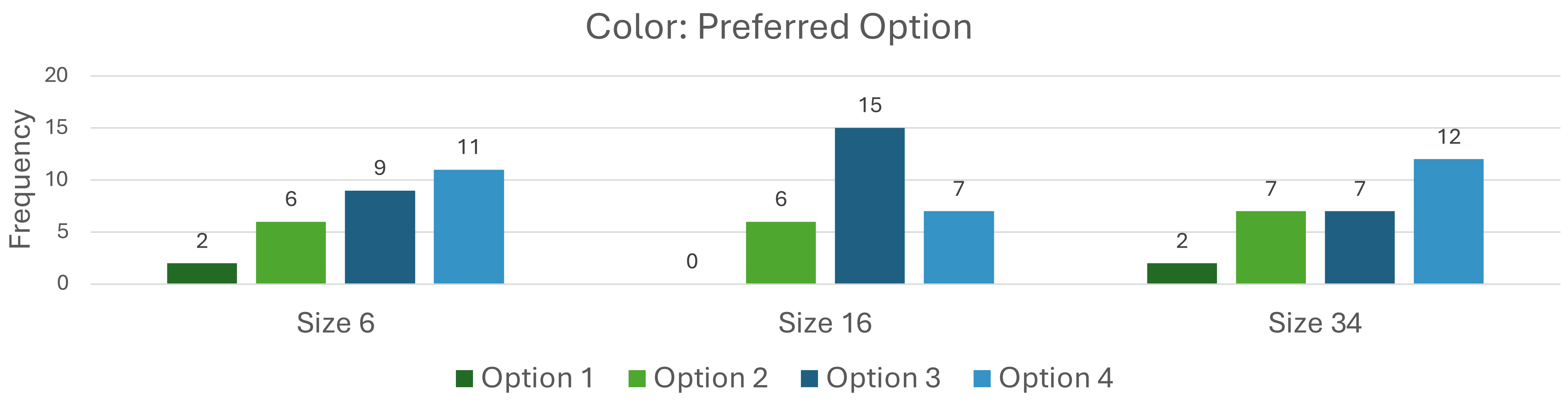
$$\varphi_7 = P(y, g(f(z, e(f(u)), h(a))), g(e(u), h(e(z))))$$

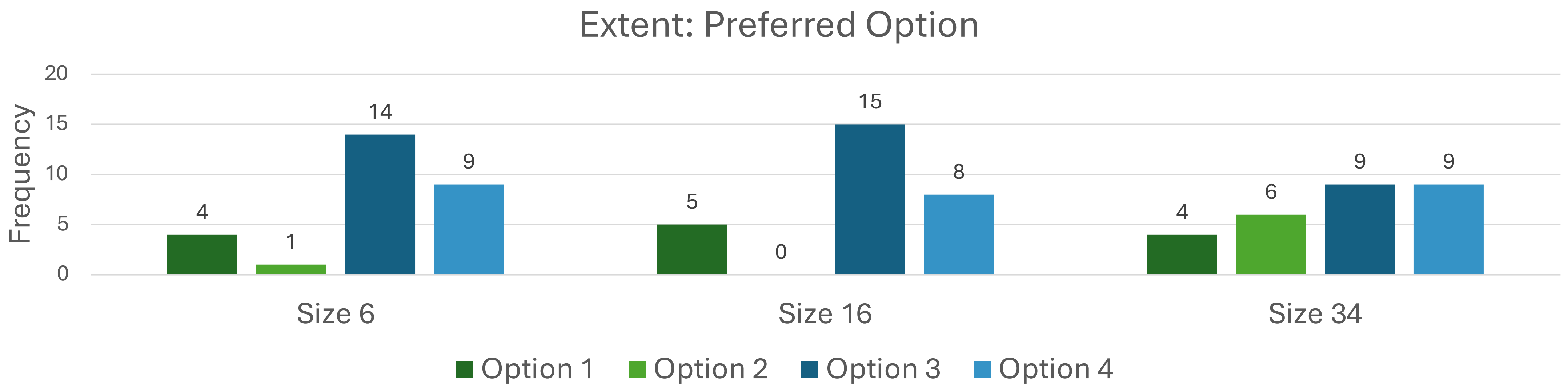
Option 4:

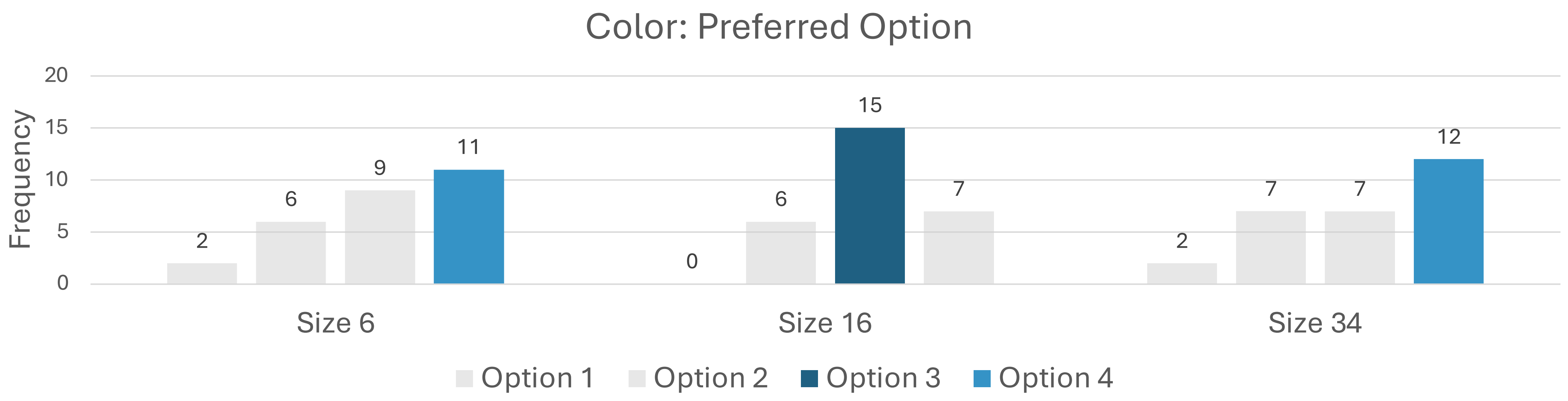
$$\varphi_7 = P(y, g(f(z, e(f(u)), h(a))), g(e(u), h(e(z))))$$

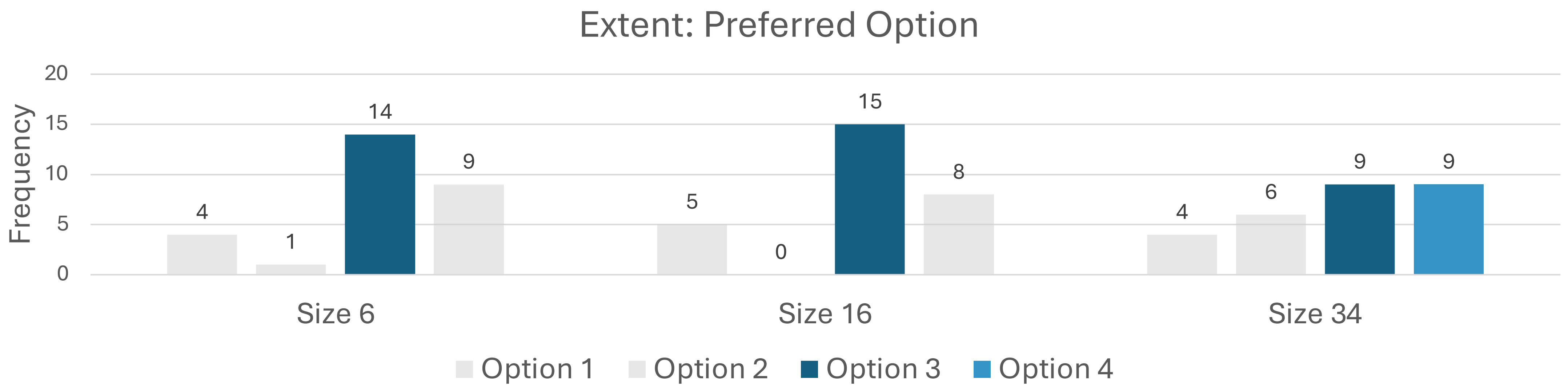
**H3** There is a threshold for how many colors can be added.

**H4** There is a threshold for how many extents can be added.


Color: Preferred Option
Frequency
20
15
10
5
0
2
6
9
11
0
6
15
7
2
7
7
12
Size 6
Size 16
Size 34
Option 1
Option 2
Option 3
Option 4

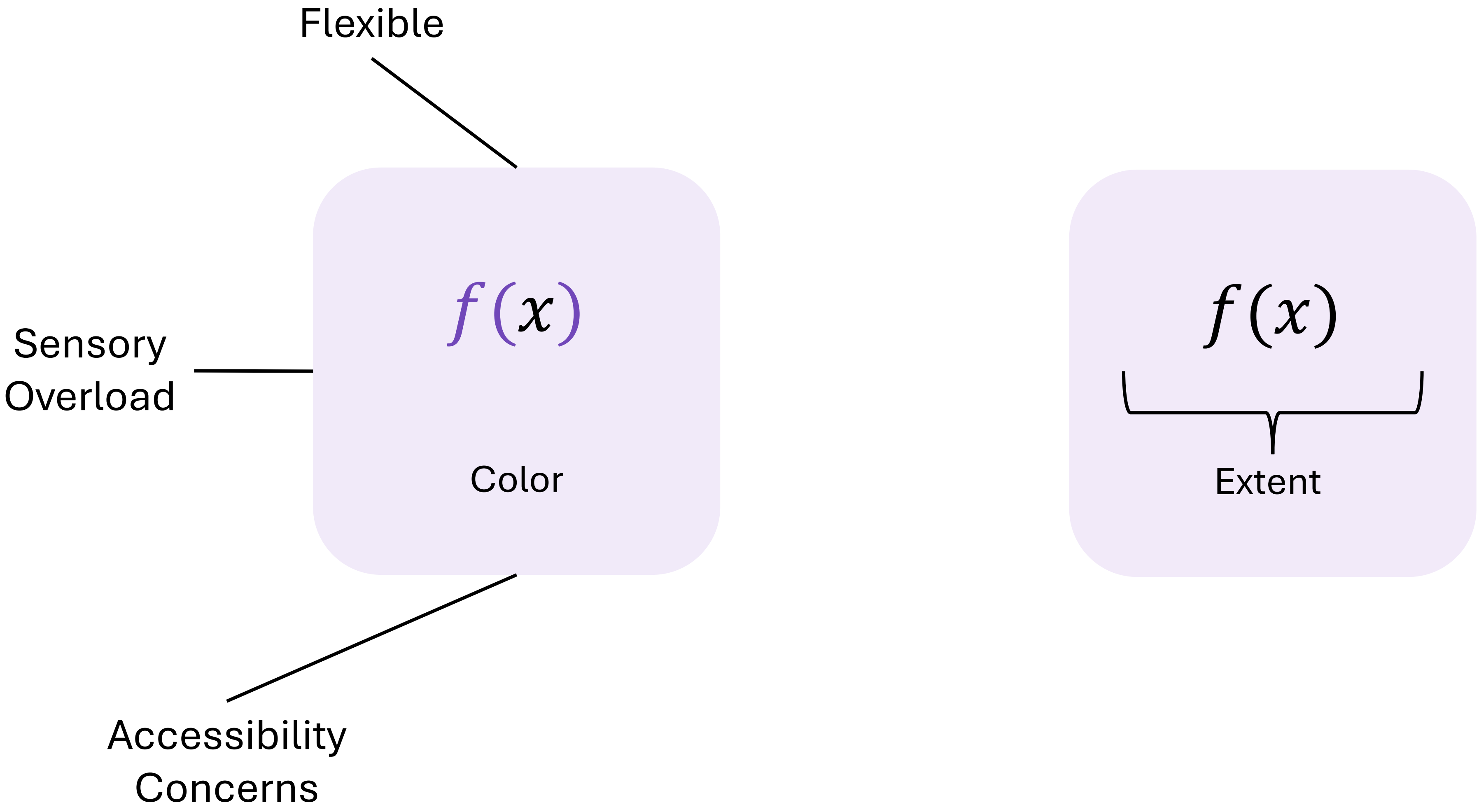

Extent: Preferred Option
Frequency
20
15
10
5
0
4
1
14
9
5
0
15
8
4
6
9
9
Size 6
Size 16
Size 34
Option 1
Option 2
Option 3
Option 4

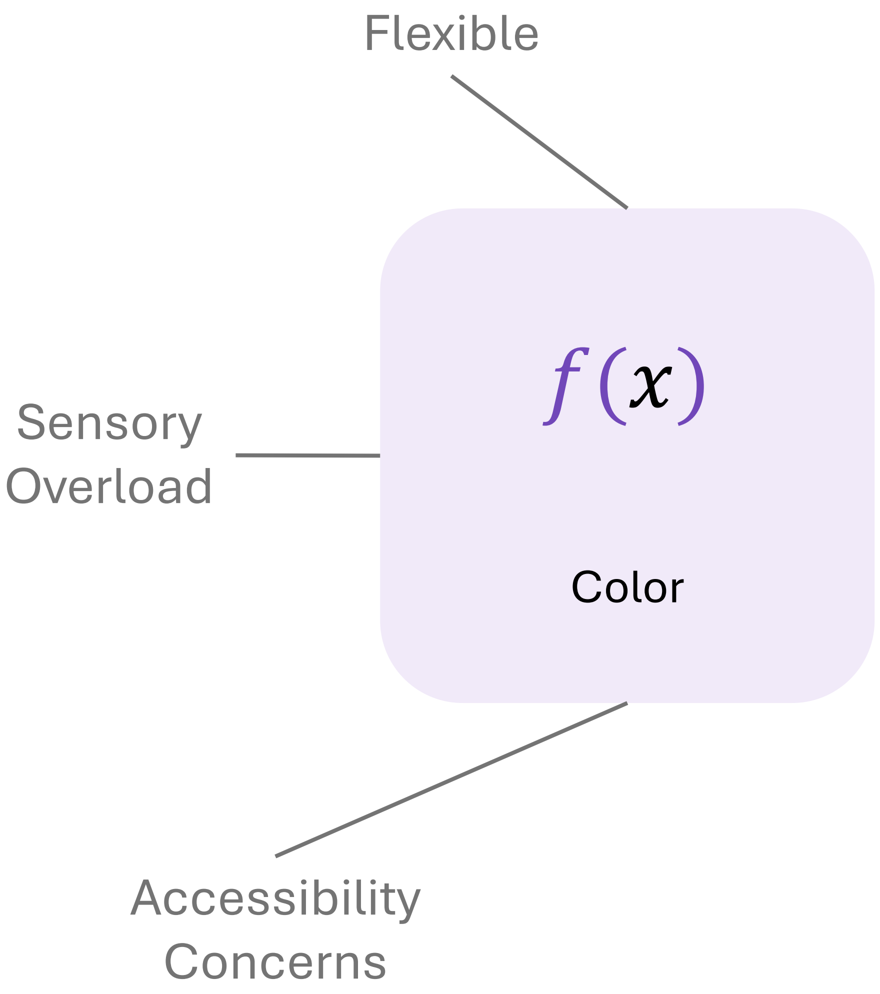

Color: Preferred Option
Frequency
20
15
10
5
0
2
6
9
11
0
6
15
7
2
7
7
12
Size 6
Size 16
Size 34
Option 1
Option 2
Option 3
Option 4

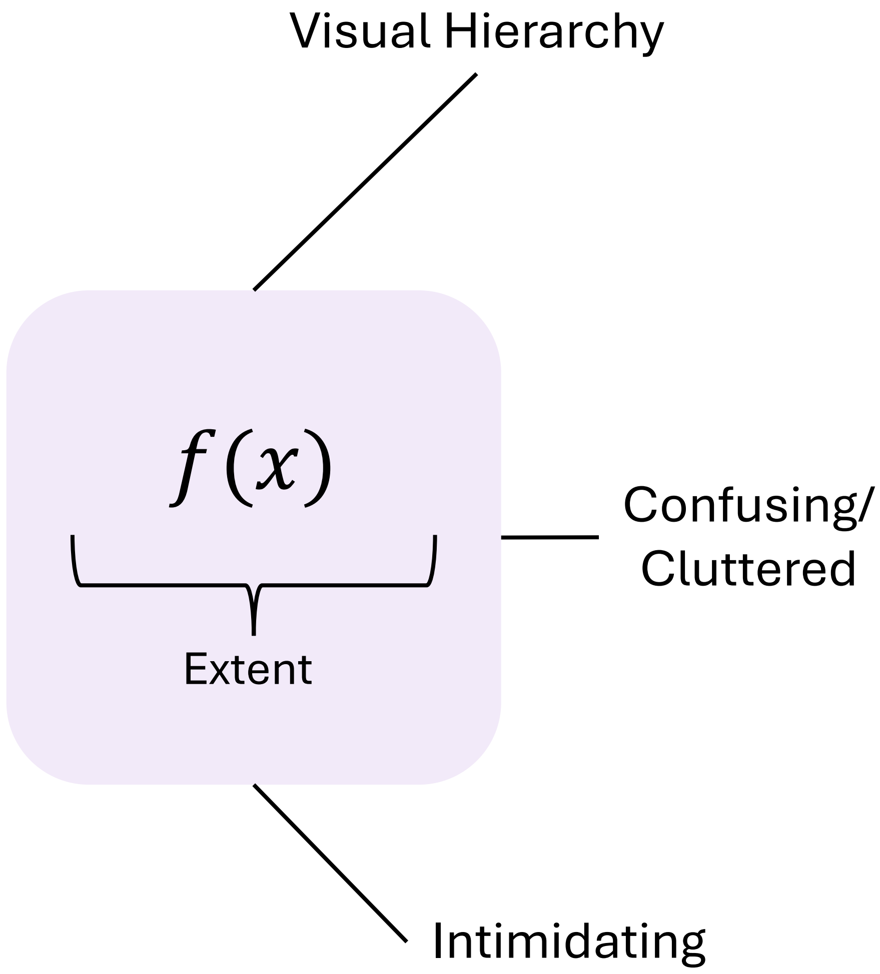

Extent: Preferred Option
Frequency
20
15
10
5
0
4
1
14
9
5
0
15
8
4
6
9
9
Size 6
Size 16
Size 34
Option 1
Option 2
Option 3
Option 4

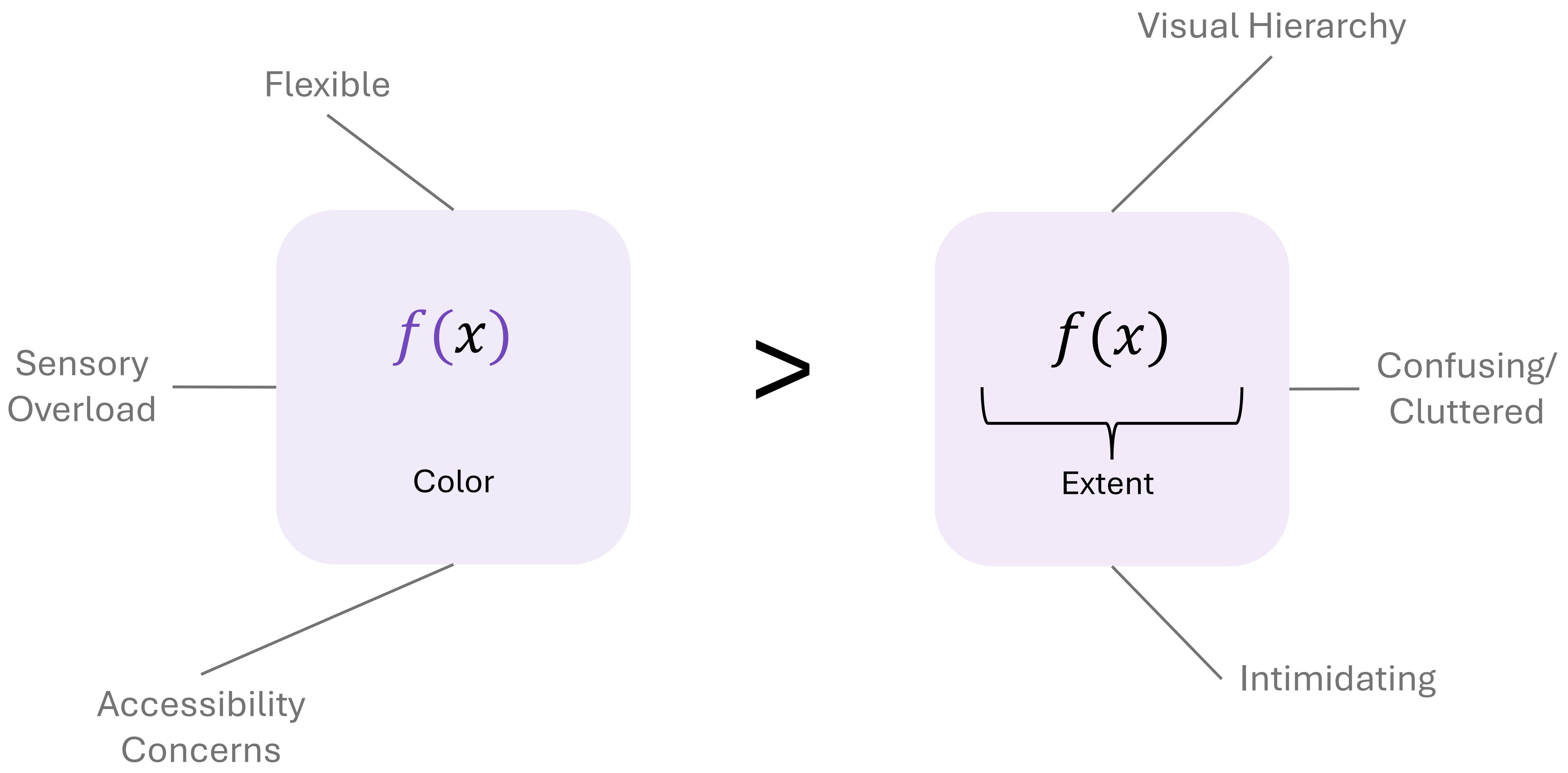
**Q.** Please share any thoughts/ comments you have about the augmentations used in this survey.

**Q.** Do you think augmentations in general can improve readability and understanding?


Flexible
Sensory Overload
$f(x)$
Color
Accessibility Concerns
$f(x)$
Extent


Flexible
Sensory Overload
$f(x)$
Color
Accessibility Concerns


Visual Hierarchy
$f(x)$
Extent
Confusing/ Cluttered
Intimidating


Flexible
Sensory Overload
Accessibility Concerns
$f(x)$
Color
>
Visual Hierarchy
Confusing/ Cluttered
Intimidating
$f(x)$
Extent

“Yes, more colors please. **Like in my IDE**.”

“Sure, but I want to stress the principle **form follows function**.”

“[...] Ideally, the **user should be able to choose** between different augmentations.”

“Yes, to a limited extend. **Too much augmentation can be confusing** [...] ”

# Discussion

# Limitations

- Do we really test Understanding?
- Ceiling Effect
- Reliability of Self-Report

# Limitations

Do we really test Understanding?

Ceiling Effect

Reliability of Self-Report

# Future Work

Other Performance Measures
Other Augmentations

Connection between Content & Augmentations

Test Augmentations in the Field with Real Work

# Take-Aways

# Take-Aways

$f(x)$ Color > $f(x)$ Extent > $f(x)$ Plain

# Take-Aways

$f(x)$ Color > $f(x)$ Extent > $f(x)$ Plain

Comprehension and as such the effect of augmentations is a difficult concept to test!

# Appendix

### 4.1.2 Positions, Size and Depth

It follows from the definitions of terms and formulas that they have a tree-like structure. For referring to a certain subtree, called subterm or subformula, respectively, sequences of natural numbers are used, called positions. The set of positions of a term/formula is inductively defined by:

- $pos(x) := \{\epsilon\}$ if $x \in X$, where $X$ is the set of variables
- $pos(\neg\varphi) := \{\epsilon\} \cup \{1p \,|\, p \in pos(\varphi)\}$
- $pos(\varphi \circ \psi) := \{\epsilon\} \cup \{1p \,|\, p \in pos(\varphi)\} \cup \{2p \,|\, p \in pos(\psi)\}$
- $pos(s = t) := \{\epsilon\} \cup \{1p \,|\, p \in pos(s)\} \cup \{2p \,|\, p \in pos(t)\}$
- $pos(f(t_1, ..., t_n)) := \{\epsilon\} \cup \bigcup_{i=1}^{n} \{ip \,|\, p \in pos(t_i)\}$
- $pos(P(t_1, ..., t_n)) := \{\epsilon\} \cup \bigcup_{i=1}^{n} \{ip \,|\, p \in pos(t_i)\}$
- $pos(\forall x \varphi) := \{\epsilon\} \cup \{1p \,|\, p \in pos(\varphi)\}$
- $pos(\exists x \varphi) := \{\epsilon\} \cup \{1p \,|\, p \in pos(\varphi)\}$

where $\circ \in \{\wedge, \vee, \rightarrow, \leftrightarrow\}$ and $t_i \in T(\Sigma, X)$ for all $i \in \{1, ..., n\}$. The size of a term $t$ (formula $\varphi$), written $|t|$ ($|\varphi|$), is the cardinality of $pos(t)$, i.e., $|t| := |pos(t)|$ ($|\varphi| := |pos(\varphi)|$). The depth of a term/formula is the maximal length of a position in the term/formula: $d(t) := \max\{|p| \,|\, p \in pos(t)\} / d(\varphi) := \max\{|p| \,|\, p \in pos(t)\}$.

Example: $pos(\neg P(f(x, g(a)))) = \{\epsilon, 1, 11, 111, 112, 1121\}$ meaning its size is 6 and its depth 4.

Arity refers to the number of arguments a function or relation takes, i.e. function and relation symbols have arity $n \in \mathbb{N}$ with $n \geq 0$. Function symbols with arity $n = 0$ are called constants. For a symbol $s(t_1, ..., t_n)$ $n$ is the arity of $s$ or $s$ is $n$-ary, e.g. $g(x, y)$ has arity 2.

In the following $Q$ is a relation symbol, $c, e, f, g, h$ are function symbols and $u, x, z$ are variables.

Give the arity (i.e. a natural number) of the function symbols $c, e, f, g, h$ and the relation symbol $Q$ for the following formula:

$$\vartheta = Q(\overbrace{\underbrace{f(\overbrace{e(z)}, g(\overbrace{\underbrace{e(\overbrace{\ c\ })}, x}))}, z, \underbrace{h(\overbrace{g(x, \underbrace{f(u, \overbrace{\ c\ })})})}})$$

Option 1:

$$\lambda_{10} = P(f(a, g(h(e(a)))), y, g(f(a, e(h(g(b))), v), h(a)), d(e(h(e(a, f(x, u, g(v, h(b))), h(e(a), z))))))$$

Option 2:

$$\lambda_{10} = P(f(a, g(h(e(a)))), y, g(f(a, e(h(g(b))), v), h(a)), d(e(h(e(a, f(x, u, g(v, h(b))), h(e(a), z))))))$$

Option 3:

$$\lambda_{10} = P(f(a, g(h(e(a)))), y, g(f(a, e(h(g(b))), v), h(a)), d(e(h(e(a, f(x, u, g(v, h(b))), h(e(a), z))))))$$

Option 4:

$$\lambda_{10} = P(f(a, g(h(e(a)))), y, g(f(a, e(h(g(b))), v), h(a)), d(e(h(e(a, f(x, u, g(v, h(b))), h(e(a), z))))))$$

Option 1:

$$\lambda_{10} = P(f(a,g(h(e(a)))),y,g(f(a,e(h(g(b))),v),h(a)),d(e(h(e(a,f(x,u,g(v,h(b))),h(e(a),z))))))$$

Option 2:

$$\lambda_{10} = P(\underbrace{\overbrace{f(a,g(h(e(a))))}, \overbrace{y}, \overbrace{g(f(a,e(h(g(b))),v),h(a))}, \overbrace{d(e(h(e(a,f(x,u,g(v,h(b))),h(e(a),z)))))}})$$

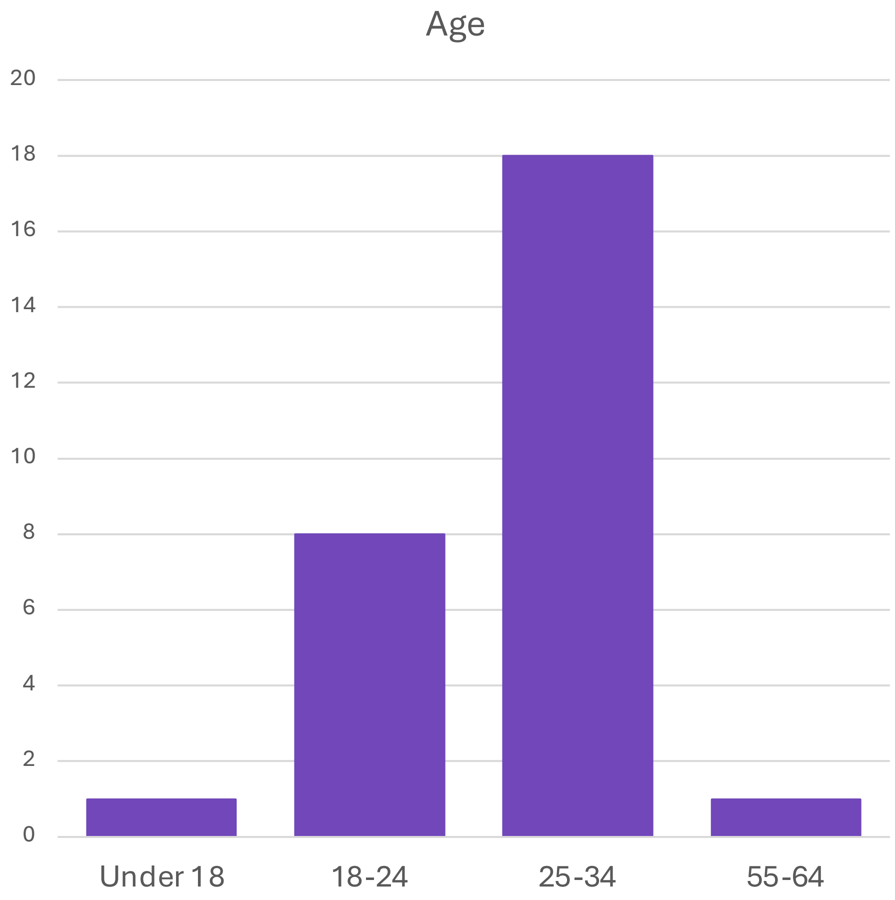
Option 3:

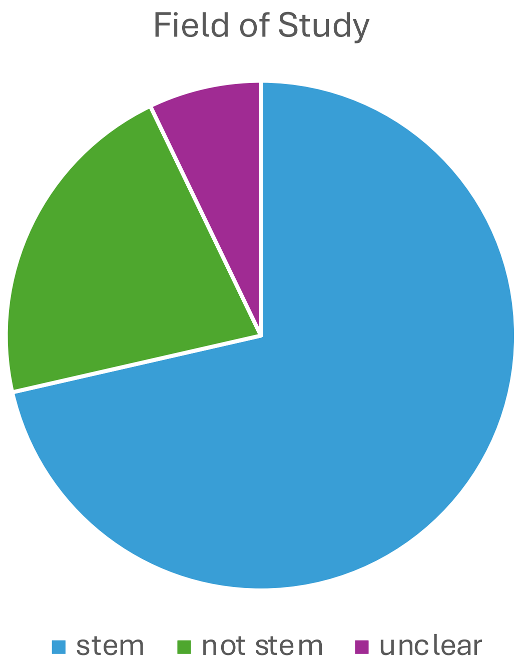
$$\lambda_{10} = P(\underbrace{\overbrace{f(\underbrace{a}, \underbrace{g(\overbrace{h(e(a))})})}, \overbrace{y}, \overbrace{g(\underbrace{f(\overbrace{a}, \overbrace{e(h(g(b)))}, \overbrace{v})}, \underbrace{h(\overbrace{a})})}, \overbrace{d(\underbrace{e(\overbrace{h(e(a,f(x,u,g(v,h(b))),h(e(a),z)))})})}})$$

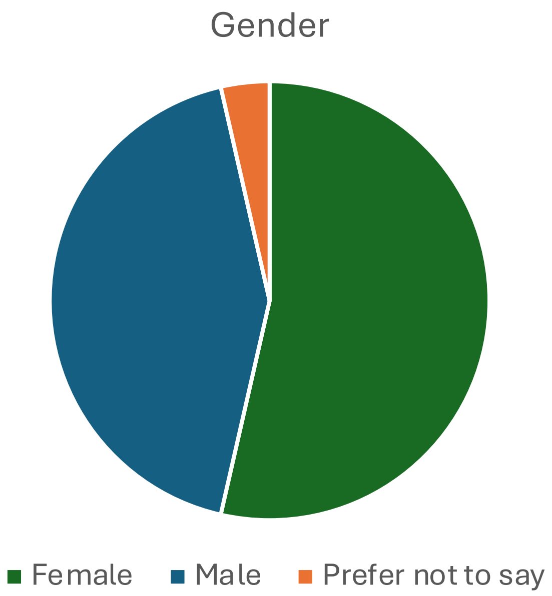
Option 4:

$$\lambda_{10} = P(\underbrace{\overbrace{f(\underbrace{a}, \underbrace{g(\overbrace{h(\underbrace{e(\overbrace{a})})})})}, \overbrace{y}, \overbrace{g(\underbrace{f(\overbrace{a}, \overbrace{e(\underbrace{h(\overbrace{g(\underbrace{b})})})}, \overbrace{v})}, \underbrace{h(\overbrace{a})})}, \overbrace{d(\underbrace{e(\overbrace{h(\underbrace{e(\overbrace{a}, \overbrace{f(\underbrace{x}, \underbrace{u}, \underbrace{g(\overbrace{v}, \overbrace{h(b)})})}, \overbrace{h(\underbrace{e(\overbrace{a})}, \underbrace{z})})})})})}})$$

# Demographics



How easy was this formula to read? *

| | 1 | 2 | 3 | 4 | 5 | |
|---|---|---|---|---|---|---|
| Very difficult | ○ | ○ | ○ | ○ | ○ | Very easy |

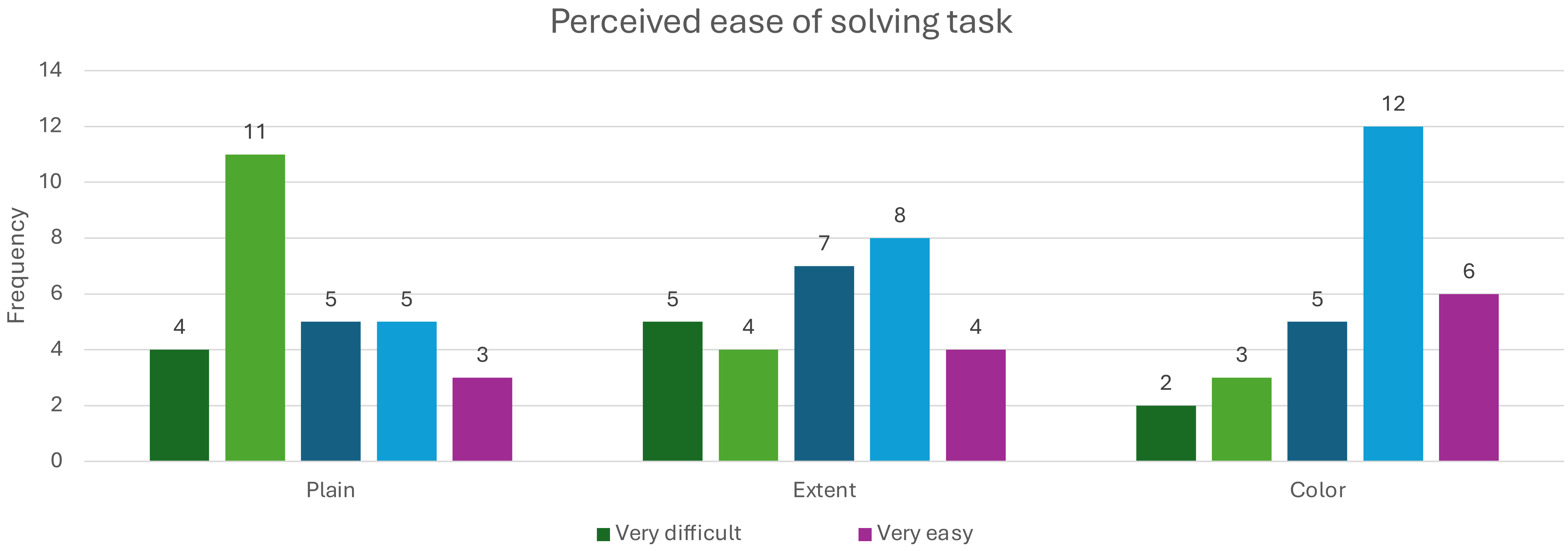
How easy was it to solve the task? *

| | 1 | 2 | 3 | 4 | 5 | |
|---|---|---|---|---|---|---|
| Very difficult | ○ | ○ | ○ | ○ | ○ | Very easy |

How helpful was the use of ...? *

| | 1 | 2 | 3 | 4 | 5 | |
|---|---|---|---|---|---|---|
| Not helpful at all | ○ | ○ | ○ | ○ | ○ | Very helpful |

# How easy was it to solve the task?




There was a **significant difference** between how easy people perceived the task to be solved ($X^2(2) = 16.8; p < 0.05$).

# How easy was it solve the task?

Post-hoc Analysis using Wilcoxon's Matched-Pairs Signed Ranks Test:

| Augmentation Type | | Direction | p-Value | T-Value |
|---|---|---|---|---|
| Plain | Color | Less | $p < 0.05$ | 0 |
| Color | Extent | Greater | $p < 0.05$ | 85.5 |

# How helpful was the use of ...?

Wilcoxon's Matched-Pairs Signed Ranks Test:

| Augmentation Type | | Direction | p-Value | T-Value |
|---|---|---|---|---|
| Color | Extent | Greater | $p < 0.05$ | 211 |

# Quiz Results

| | Plain | Color | Extent |
|---|---|---|---|
| **Correct** | 0.83% | 0.86% | 0.80% |
| **Correct*** | 0.89% | 0.93% | 0.85% |

* Excluded participants that answered only “x”

# References


- Kislenko, K., Grevholm, B., & Lepik, M. (2007). Mathematics is important but boring : students' beliefs and attitudes towards mathematics. Relating Practice and Research in Mathematics Education : Proceedings of NORMA 05, Fourth Nordic Conference on Mathematics Education, Trondheim, 2nd-6th September 2005, 349–360.
- Waswa, D.W., Al-kassab, M.M. (2023). Mathematics Learning Challenges and Difficulties: A Students' Perspective. In: Zeidan, D., Cortés, J.C., Burqan, A., Qazza, A., Merker, J., Gharib, G. (eds) Mathematics and Computation. IACMC 2022. Springer Proceedings in Mathematics & Statistics, vol 418. Springer, Singapore.
- Head, Andrew, et al. "Math augmentation: How authors enhance the readability of formulas using novel visual design practices." *Proceedings of the 2022 CHI Conference on Human Factors in Computing Systems*. 2022.